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EFFECTIVE METHOD OF CALCULATING THE WAVE DRAG OF SOLIDS OF REVOLUTION IN THE TRANSONIC RANGE*

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The determination of integral aerohydrodynamic characteristics is one of the principal objectives in the solution of aerodynamic problems. Here, we describe an effective new algorithm for calculating the wave drag of solids of revolution in a transonic flow of an ideal gas. The calculations are performed within the framework of the theory of small perturbations.

In the numerical study of problems concerning flow about bodies, the necessary aerodynamic coefficients are usually calculated by integrating the distribution of pressure over the surface of the body. However, it should be noted that the accuracy of this approach may be affected by errors made in the numerical calculation — particularly in the nose and tail regions. Calculations have shown that these errors are negligible in the determination of the lift and moment coefficients and that the traditional method is acceptable for both steady and unsteady problems.

The situation is different in regard to determination of the wave drag. It is known [1, 2] that integration errors — connected both with errors in the numerical analysis and with violations of the postulates of the transonic theory of small perturbations at individual points on the body — may produce errors in the final result obtained by the traditional approach. These errors may even lead to negative values for the drag coefficient. Thus, here we use another method to determine the wave drag of bodies — a method that is less sensitive to integration errors. The authors of [2] were the first to propose replacing integration of pressure over the surface of the body by another procedure that does not involve integration at locations where the postulates of the transonic theory may be violated. The alternative method has now been used extensively in studies devoted to finding ways to determine steady and unsteady wave drag in transonic flows.

For example, it was proposed in [1, 2] that wave drag in a steady flow past thin airfoils be found by integrating along shock waves enveloping supersonic regions. This method is less sensitive to numerical integration errors connected with specific features of perturbation theory. Also, as was noted in [1], in the given regimes viscosity affects the position and intensity of the shock wave only in its lower region. The upper part, formed by superimposed compression waves, is independent of the interaction of the wave and the boundary layer. Local errors due to deviations from the theory of small perturbations (such as those occurring in the vicinity of the blunt nose of the body) also have little affect on the position and intensity of shocks enclosing supersonic regions.

A similar approach to determining steady-state wave drag was used in [3], where it was assumed that the contour integral of the longitudinal component of momentum along the shocks is equal to the contour integral over the surface of the body.

The author of [4] and [5] generalized the method to the case of unsteady transonic flow and obtained the time dependence of aerodynamic characteristics of wing profiles for different transients — such as the interactions of the profile with a wind shear, a moving shock wave, etc.

The values of drag calculated in the above-cited works for different airfoils agree well with the experimental data, which shows the effectiveness and reliability of the method. It is interesting to attempt to extend the method to the case of flow about axisymmetric bodies. There are certain features specific to this problem which have not been examined in the literature. Here, we obtain a drag formula for solids of revolution in steady transonic flow, develop a numerical algorithm to calculate transonic flow about solids of revolution (the axisymmetric analog of the method of variable directions, with the Engquist–Osher monotone algorithm), and compare numerical calculations of the wave drag of certain solids of revolution with empirical data. Using the transonic rule of equivalence, numerical results are obtained for the resistance of a complex

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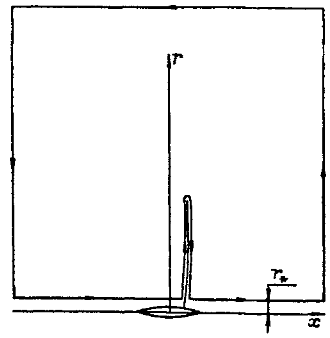

Fig. 1

three-dimensional aircraft configuration of elongated form. The results are obtained for the transonic velocity range and are compared with experimental data.

1. Formulation of the Problem. The problem of transonic flow past a prolate solid of revolution can be examined within the framework of the nonlinear transonic theory of small perturbations. Here, we obtain steady-state solutions by proceeding on the basis of the equation for unsteady conditions. We write this equation as follows in dimensionless form

$$A\,\varphi_{xt} = B\,\varphi_{xx} + \frac{1}{r}(r\varphi_r)_r$$
$$(A = 2\,M_\infty^2,\ \ B = 1 - M_\infty^2 - (\gamma+1)\,M_\infty^2\,\varphi_x). \qquad (1.1)$$

Here, φ is the potential of perturbed velocity, referred to the velocity of the incoming flow U_∞ and the length of the body in the flow L; the independent state variables (the longitudinal coordinate x and the transverse coordinate r) are referred to L; time t is referred to L/U_∞; γ is the adiabatic exponent; M_∞ is the Mach number of the undisturbed flow.

As the boundary conditions at infinity for Eq. (1.1), we use the so-called nonreflecting conditions proposed in [6] for plane transonic flow. The derivation of these relations is based on analysis of the characteristic equation obtained from the equations of the transonic theory of small perturbations. Since the characteristic equation for the axisymmetric case is identical in form to the analogous equation for the plane case, we can use the results in [6] to obtain asymptotic relations for large distances in the case of axial symmetry:

$$\varphi_t - B\,\varphi_x/A = 0,\ \ x \to -\infty;\ \ \varphi_x = 0,\ \ x \to \infty;\ \ \varphi_r + \sqrt{B}\varphi_x = 0,\ \ r \to \infty.$$

The boundary condition for the body requires special consideration. Since the initial equation contains a singularity in the cylindrical coordinate system at r = 0, it is incorrect to take the boundary condition on the axis of symmetry. Instead, it should be assigned on a certain cylindrical surface of small radius that separates the entire region of the solution into external and internal parts. Then the resulting solutions are joined to obtain the complete solution.

It is known [7] that flow near an axisymmetric body is described by the following equation in the internal region

$$(r\varphi_r)_r = 0.$$

Having integrated this equation once, we obtain

$$r\varphi_r = f(x), \qquad (1.2)$$

where f(x) is an arbitrary function. We can find this function by using the condition of nonflow on the surface of the body $\varphi_r = R'(x)$ (R(x) is the form of the solid of revolution). Thus,

$$f(x) = RR'(x).$$

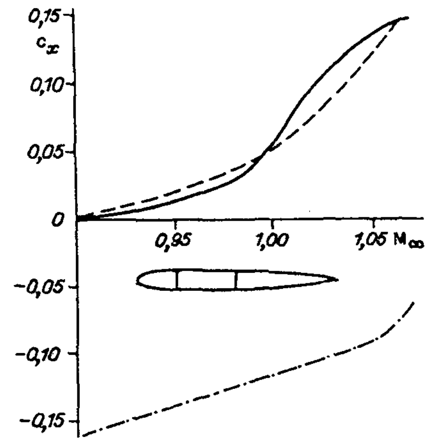

Fig. 2

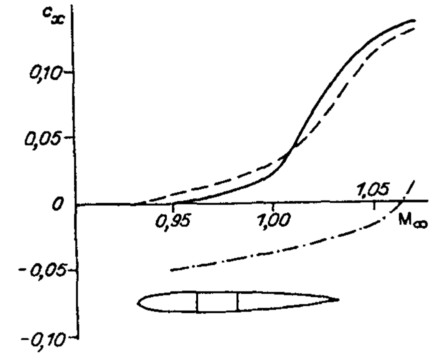

Fig. 3

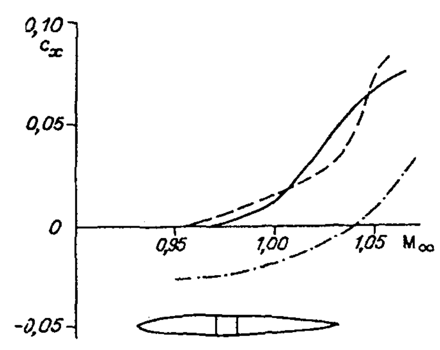

Fig. 4

Finally, we write the boundary condition for the external region in the form

$$\lim_{r \to 0} r \varphi_r = \frac{S'(x)}{2\pi}$$

(S(x) is the cross-sectional area of the body in the flow).

In the numerical solution of the problem, the given relation is assigned for the surface of a cylinder of sufficiently small radius r_*. The specific value of r_* determines the boundary where the solutions are joined and should be chosen on the basis of the condition that its further decrease will have almost no effect on the pressure distribution on the body obtained by solving the boundary-value problem. A procedure for choosing the optimum value in relation to the Mach number of the incoming flow and the thickness of the body was described in [8, 9].

2. Method of Calculating Wave Drag. Certain transformations must be performed before the integral theorem of momenta can be used to calculate the wave drag of a solid of revolution by integrating over a shock wave. The transformations are necessary because the expression for pressure on the body in the axisymmetric case differs from the analogous expression for the plane case within the context of the theory of small perturbations. In addition, as in the plane case, the original integration method is unsuitable because of certain features in the axial structure of the flow.

With the assumption that the perturbations are small, we write the expression for the pressure coefficient in the form

$$c_p = -2u - v^2.$$

The transverse component of velocity on the body is known from the boundary conditions. To determine the longitudinal component, we once again integrate Eq. (1.2):

$$\varphi = \frac{S'_x}{2\pi} \ln r + g(x).$$

Here, g(x) is a function determined by joining the result with the solution of the external problem. For the longitudinal component of velocity we obtain

$$u = \varphi_x = \frac{S''_{xx}}{2\pi} \ln r + g'_x. \tag{2.1}$$

Since the boundary-value problem has been solved, i.e. since the field of perturbed velocity is known, then we also know the distribution of the longitudinal component u_* on the surface of the imaginary cylinder at $r = r_*$. Combining the internal and external solutions at $r = r_*$, we find the value of g' in (2.1) and, thus, the sought distribution of the longitudinal component of velocity on the surface of the body:

$$u(x, R) = \frac{S''_{xx}}{2\pi} \ln \frac{R}{r_*} + u_*.$$

Now we can determine the wave-drag coefficient of the body as the integral of pressure on the body:

$$c_x = -\int_{-0,5}^{0,5} 2\pi R R'_x \left(2u_* + \frac{S''_{xx}}{\pi} \ln \frac{R}{r_*} + (R'_x)^2\right) dx.$$

We take condition (1.2) into account and write this expression in the form of the sum of two integrals:

$$c_x = -4\pi \int_{-0,5}^{0,5} r_* u_* v_* \, dx - 2\pi \int_{-0,5}^{0,5} \left(\frac{S''_{xx}}{\pi} \ln \frac{R}{r_*} + (R'_x)^2\right) R R'_x \, dx. \tag{2.2}$$

The second term can be integrated:

$$2\pi \int_{-0,5}^{0,5} \left(\frac{S''_{xx}}{\pi} \ln \frac{R}{r_*} + (R'_x)^2\right) R R'_x \, dx = \frac{(S'_x)^2}{2\pi} \ln \frac{R}{r_*} \Bigg|_{x=-0,5}^{x=0,5}. \tag{2.3}$$

Since only thin and (as a rule) closed bodies are examined within the framework of the transonic theory of small perturbations, the terms of (2.3) vanish. Specifically, this occurs because the limit $R^2 \ln R \to 0$ as $R \to 0$ in the nose and tail portions of the body. In the case of a body with a nontrivial radius in the bottom part, the complement of resistance — the term with $x = 0.5$ — may not be zero. In this case, it can be calculated in accordance with (2.3). Thus, the first term remains unknown in the expression for drag (2.2) and must be found.

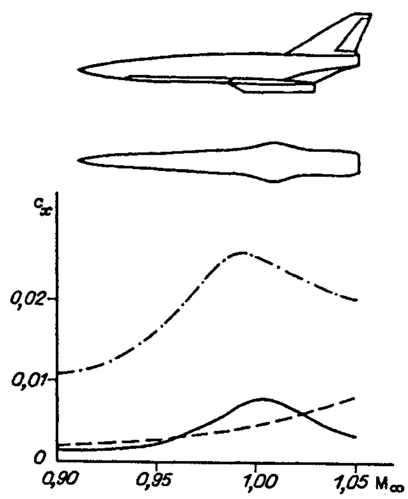

Fig. 5

As already noted, certain errors may produce incorrect results when the wave drag of a body is calculated by integrating pressure over the body. Negative values may even be obtained. We will attempt to avoid such an outcome by using an approach described in [2] for the plane case. We will examine a system consisting of the stationary analog of Eq. (1.1) and an equation expressing the condition for the absence of vorticity in the flow:

$$r(1-\mathrm{M}_\infty^2)u_x - r(\gamma+1)\mathrm{M}_\infty^2\, u\, u_x^2 + (rv)_r = 0,$$
$$u_r - v_x = 0.$$

Multiplying the first equation of the system by u and the second by rv and adding the results, we obtain a relation having a divergent form:

$$\Big(r\Big(\frac{1-\mathrm{M}_\infty^2}{2}\,u^2 - \frac{\gamma+1}{3}\,\mathrm{M}_\infty^2\,u^3 - \frac{v^2}{2}\Big)\Big)_x + (r\,u\,v)_r = 0.$$

Taking the double integral of this expression over the entire theoretical flow region and using Green's formula to reduce it to a curvilinear integral over a closed contour (Fig. 1), we find that

$$\oint r\Big(\frac{1-\mathrm{M}_\infty^2}{2}\,u^2 - \frac{\gamma+1}{3}\,\mathrm{M}_\infty^2\,u^3 - \frac{v^2}{2}\Big)\,dr - r\,u\,v\,dx = 0. \tag{2.4}$$

At $\mathrm{M}_\infty \leq 1$, the asymptote of the long-range flow field is such [1] that the integrals over the external boundaries approach zero as the boundaries of the theoretical region approach infinity. Thus, we can write (2.4) in the form

$$\int_s r\Big[\frac{1-\mathrm{M}_\infty^2}{2}\,u^2 - \frac{\gamma+1}{3}\,\mathrm{M}_\infty^2\,u^3 - \frac{v^2}{2}\Big]\,dr - \int_s r\,[u\,v]\,dx - \int_{-0,5}^{0,5} u_*\,v_*\,r_*\,dx = 0,$$

where the index s denotes integration along a shock wave or several shock waves. The brackets denote that the enclosed quantity undergoes a discontinuity in the transition through the shock wave. The resulting expression means that, to within the multiplier 4π, the first term in Eq. (2.2) can be expressed in terms of integrals over a shock wave. Here, the integrands differ from the corresponding expressions for the plane case [1] only in the presence of the independent coordinate r, which is continuous in the transition through the shock. Omitting the intermediate calculations (which are analogous to the calculations performed in [1]), we present the final result in the form

$$\int_{-0,5}^{0,5} u_* v_* r_* \, dx = -M_\infty^2 \frac{\gamma+1}{12} \int_s r\,[u]^3\, dr,$$

so that

$$c_x = 4\pi \frac{\gamma+1}{12} M_\infty^2 \int_s r\,[u]^3\, dr. \tag{2.5}$$

In contrast to the first integral in Eq. (2.2) — the value of which represents a small difference between two similar quantities and thus contains unacceptably large errors (as already noted, the result may even turn out to be negative), integral (2.5) is always positive, since the integrand is positive due to the physical nature of the problem.

3. Numerical Method and Results of Calculation. The boundary-value problem formulated for Eq. (1.1) was solved numerically by the steady-state method. To obtain the solution, we devised an axisymmetric variant of the implicit method of variable directions and used the Engquist–Osher monotone algorithm. This approach was used in [10-12] to calculate plane unsteady transonic flows. Using the method of variable directions allows us to break the problem down into two stages. In the first stage, we pass over all horizontal lines of the computing grid in the x-direction and calculate an intermediate value of potential. We then use this value in the second stage (passing over all vertical lines in the r-direction) to obtain values of potential for the next time layer. It should be noted that this is not an iterative method. The value of potential for the next time layer is calculated by direct use of the value of potential for the preceding layer, expressed in terms of the intermediate potential. The calculation entails the solution of a system of linear equations. The differential equation is nonlinear.

Despite the fact that we are seeking the steady-state solution of Eq. (1.1), the boundary condition for the left boundary contains an unsteady term. However, having such a boundary condition means that a perturbation from the body that reaches the inlet boundary will not be reflected back into the theoretical region. This in turn makes it possible to obtain the steady-state solution more quickly.

Numerical calculations were performed on a rectangular grid consisting of 121 nodes in the x-direction and 81 nodes in the r-direction. The grid was made denser near the forward and rear edges and sparser with approach of the external boundaries of the theoretical region — 30 body-lengths distant in each direction. There were 81 nodes on the body, which was positioned symmetrically relative to the x axis ($|x| \le 0.5$).

The potential field from the near-range variant was usually used as the initial conditions. In this case, the new solution was obtained after 120-150 iterations. To obtain the first variant, we assigned a zero initial potential field and gradually increased the Mach number to a specific transonic value. In this case, the steady-state solution was obtained after 180-200 iterations.

To calculate drag by the method of integrating over shock waves, we developed an algorithm to locate the waves on the basis of abrupt reductions in the local supersonic Mach number in the flow field. The algorithm was used to calculate the steady wave drag of solids of revolution of simple form. Using the transonic rule of equivalence, we also used the algorithm to calculate the wave drag of a schematized hypersonic aircraft of relatively complex configuration.

We will present the results of numerical calculations of the steady wave drag of three solids of revolution with a relative thickness of 10%. The calculations were performed for the transonic range of Mach numbers M_∞ from 0.9 to 1.07. Each body consisted of three parts: the nose, which was close to an ellipsoid of revolution; the central part, of cylindrical form; the spindle-shaped tail. The tail portion of all three bodies was the same and accounted for half their length. The nose accounted for 20, 30, and 40% of the entire lengths of the first, second, and third bodies, respectively. The nose and central part were smoothly joined, as were the central part and the tail, i.e., the derivative of the function R(x) was continuous at their junctions. Flow about the given bodies was studied experimentally in [13], which described their geometries and presented data on wave drag.

Figures 2-4 show the bodies and the dependences of wave drag on the Mach number: the solid curve shows the numerical result, while the dashed line shows the experimental data [13]. The good agreement of the two sets of data is evident. It should be noted that the numerical algorithm makes it possible to calculate not only subsonic regimes $M_\infty < 1$, but also supersonic regimes with values of M_∞ slightly greater than unity. Accordingly, bow waves or attached shocks form ahead of the bodies, and the algorithm must include integration along these waves as well.

The dot−dash curves in Figs. 2-4 show the result of calculation of drag by integrating the pressure distribution obtained from numerical solution of the boundary-value problem formulated for Eq. (1.1). It is evident that these results differ significantly from not only the experimental data, but also the results obtained by integration along the shocks; in addition, they are physically incorrect within a fairly large range of values of M_∞. This result is consistent with the findings in [2], where the author also obtained negative values of wave drag for airfoils in integrating numerical values of pressure distribution over their contours.

The method developed here was used together with the transonic equivalence rule to calculate the wave drag of a schematized hypersonic aircraft. In accordance with this rule, the wave drag of a prolate body of complex configuration coincides in a first approximation with the drag of a body having the same distribution of cross-sectional area. Figure 5 shows the schematized aircraft and the corresponding equivalent solid of revolution, which has a tail section. Since flow about the bottom part of the body cannot be studied by the methods of small perturbation theory, we examined the structure as a semi-infinite body with a cylindrical sting of constant radius. The pressure acting on the bottom part was taken equal to the static pressure in the incoming flow. Wind-tunnel test data obtained with different conditions for the bottom part (different sting designs and dimensions, etc.) are usually referred to a standard bottom pressure by special methods, this pressure being equal to the static pressure at the inlet to the working part of the tunnel.

In calculating the resistance of the equivalent solid of revolution, we considered that the radius of the body in the tail section $R \neq 0$ at $x = 0.5$ and that the second term in Eq. (2.2) was nontrivial. However, our calculations showed that it is two orders smaller than the first term.

Figure 5 shows the dependence of wave drag on the Mach number of the aircraft and the equivalent solid of revolution. The dashed experimental curve was obtained at TsAGI (Central Aerohydrodynamic Institute) and corresponds to the wave drag of the aircraft model calculated as the difference between the total drag for the specified Mach number and the drag at $M_\infty = 0.7$ for subcritical flow. The dashed curve corresponds to the drag calculated by the method of integration over the body. It is apparent that the values of wave drag calculated by the method of integration over a shock wave (solid curve) are considerably closer to the experimental data than the values obtained by integrating over the body. We should point out that the aircraft model being discussed is fairly complex, including an engine nacelle and corresponding to a thick equivalent solid of revolution (thickness 14%). This is probably the source of error in the theoretical curve. In particular, the complexity of the model is probably responsible for the location of the drag maximum in the supersonic region.

REFERENCES

1. J. Cole and L. Cook, Transonic Aerodynamics [Russian translation], Mir, Moscow (1989).
2. E. M. Murman and J. D. Cole, "Inviscid drag at transonic speeds," AIAA Pap., No. 74-0540 (1974).
3. J. L. Steger and B. S. Baldwin, "Shock waves and drag in the numerical calculation of compressible, irrotational transonic flow," AIAA J., **11**, No. 7 (1973).
4. A. S. Fonarev, "Aerodynamics of wing airfoils in nonstationary transonic flows," Russ. J. Comput. Mech., No. 4 (1993).
5. A. S. Fonarev, "Airfoil in a transonic flow with wind shear and weak shock waves," Prikl. Mekh. Tekh. Fiz., **34**, No. 3 (1993).
6. D. Kwak, "Non-reflecting far-field boundary conditions for unsteady transonic flow computation," AIAA J., **19**, No. 11 (1981).
7. H. Eshly and M. Lendal, Aerodynamics of Aircraft Wings and Fuselages [Russian translation], Mashinostroenie, Moscow (1969).
8. J. A. Krupp and E. M. Murman, "The numerical calculation of steady transonic flow past thin airfoils and slender bodies," AIAA Pap., No. 71-0556 (1971).
9. I. V. Tret'yakova and A. S. Fonarev, "Effect of the permeable boundaries of a transonic flow on the flow about solids of revolution," Uch. Zap. TsAGI, **9**, No. 6 (1978).
10. W. F. Ballhaus and P. M. Goorjian, "Implicit finite-difference computations of unsteady transonic flows about airfoils," AIAA J., **15**, No. 12 (1977).
11. P. M. Goorjian, M. E. Meagher, and R. V. Buskirk, "Monotone switches in implicit algorithms for potential equations applied to transonic flows," AIAA J., **23**, No. 4 (1985).

12. P. M. Goorjian and R. V. Buskirk, "Implicit calculations of transonic flow using monotone methods," AIAA Pap., No. 81-0331 (1981).
13. K. P. Petrov, Aerodynamics of Aircraft Elements [in Russian], Mashinostroenie, Moscow (1985).